Short-range two-particle correlations from statistical clusters

A.Bialas[a] and A.Bzdak[b]

[a]M.Smoluchowski Institute of Physics
Jagellonian University*
[b]AGH University of Science and Technology
Faculty of Physics and Applied Computer Science†

May 20, 2016

Keywords: two-particle correlations, statistical clusters
PACS: 12.40.Ee, 13.85.Hd, 13.87.Fh

Abstract

The two-particle short-range correlation functions in rapidity, azimuthal angle and transverse momentum, following from the decay of statistical clusters are evaluated and discussed.

1 Introduction

Studies of correlations between particles produced in high-energy collisions is a well-known method to investigate the dynamics of the production process [1]. They are conveniently divided into "short-range" when the momenta of the studied particles are close to each other and "long-range", extending over large distances in momentum space.

Already in early seventies, studies of the short-range correlations in rapidity led to the discovery that particle production proceeds through production

*Address: Lojasiewicza 11, 30-348 Krakow, Poland; e-mail: bialas@th.if.uj.edu.pl
†Address: al. Mickiewicza 30, 30-059 Krakow, Poland; e-mail: bzdak@fis.agh.edu.pl

of "clusters" [2]. This was later confirmed by more detailed studies in other variables [1], although the very nature of these clusters is unclear even now. The problem is of importance because it touches the mechanism of hadronisation, i.e., transition from the parton system, created at the early stage of the collision, into the produced hadrons. This apparently non-perturbative transition, essential to derive the structure of the produced parton system from the observed hadrons, cannot be easily treated by theory. Thus a phenomenological analysis is needed.

An interesting approach to this problem was formulated in the statistical cluster model [3, 4] which assumes that the transition from the early state of the process of particle production, dominated by parton interactions, proceeds through an intermediate stage of clusters emitting (isotropically) the final hadrons according to the rules of statistical physics[1].

The decay distribution of such a statistical cluster at rest is taken in the form of the Boltzmann distribution, which for a cluster moving with the four-velocity u^μ becomes

$$dN_1(p;u) \sim e^{-\beta p_\mu u^\mu} d^2p_\perp dy, \tag{1}$$

where $\beta = 1/T$ is the inverse cluster temperature and $p_\perp$ and y are the transverse momentum and rapidity of the final particle.

Although the model was originally constructed for description of the "soft" processes (involving only small transverse momenta), it remains an interesting question to what extent it is also applicable to semi-hard and perhaps even hard collisions. Indeed, the patron-hadron transition being a soft process, happening at the very end of the parton cascade, may very well be universal, i.e., (quasi)independent of the mechanism of parton production. If this is actually the case, the statistical clusters should be visible also at higher transverse momenta and perhaps even in all processes of particle production at high energies. This attractive possibility was recently supported by the evaluation [7] of the transverse momentum spectrum of the produced charged hadrons. It turned out that if the distribution of the cluster transverse Lorentz factor $\gamma_\perp$ ($\gamma_\perp^2 = 1+u_\perp^2$, where $u_\perp$ is the transverse component of the cluster four-velocity) follows a simple power law $\sim \gamma_\perp^{-\kappa}$ then, surprisingly

[1] This is an attractive modification of the standard statistical model, because it allows to explain the observed anisotropy of the momentum spectra of particles produced in high-energy collisions (which is a difficulty for the statistical model), while keeping, at the same time, its successes in description of particle abundances [5, 6].

enough, the transverse momentum distribution of the emitted particles

$$\frac{dN_1(p_\perp)}{dp_\perp^2} \sim \int \frac{d\gamma_\perp}{\gamma_\perp^\kappa} K_0(\beta\gamma_\perp m_\perp) I_0(\beta u_\perp p_\perp) \tag{2}$$

closely resembles the Tsallis formula [8] which, as is well-known [9–11], closely resembles the shape of the data [12–15]. See [7] for more details and [16] for further discussion.

This apparent success of the concept of the statistical cluster invites one to study its other consequences, particularly those which may provide more demanding tests of the idea. Following this route, in the present paper we discuss the two-particle correlations and show that, indeed, they give strong constraints on the model and, eventually, can be even used to pin down possible inter-cluster correlations[2].

To determine T and κ, the only free parameters of the model, we have fitted the cluster formula (2) to the transverse momentum distribution of pions and kaons produced in 2.76 TeV proton-proton (p+p) collisions [17]. The fit gives $\kappa = 5$ and the cluster temperature $T = 140$ MeV. The result is shown in Fig. 1, where one sees that the model reproduces the data with better than 20% accuracy, which is good enough for our purpose. It is also remarkable that the same power-law distribution (without change of normalization) describes well both pion and kaon distributions. We also checked that the model describes the charged particle spectra up to $p_\perp = 200$ GeV in p+p at $\sqrt{s} = 7$ TeV [18].

In the next section the general formula for the two-particle correlations in the statistical cluster model is written down, and correlations in rapidity, azimuthal angle and in transverse momentum are derived. Our results are described in Section 3. Summary and comments are given in the last section.

2 Two-particle correlations

The two-particle distribution is the sum of the contribution from one cluster and that from two different clusters $dN_2 = dN_2^{(1c)} + dN_2^{(2c)}$. Ignoring correlations in cluster decay (see section 4 for further discussion) we have

$$dN_2^{(1c)}(p_1, p_2) = \int du W(u) dN_1(p_1; u) dN_1(p_2; u) \tag{3}$$

[2]To our knowledge, the first application of the statistical model for description of cluster decays was proposed by Hayot, Henyey and Le Bellac [2].

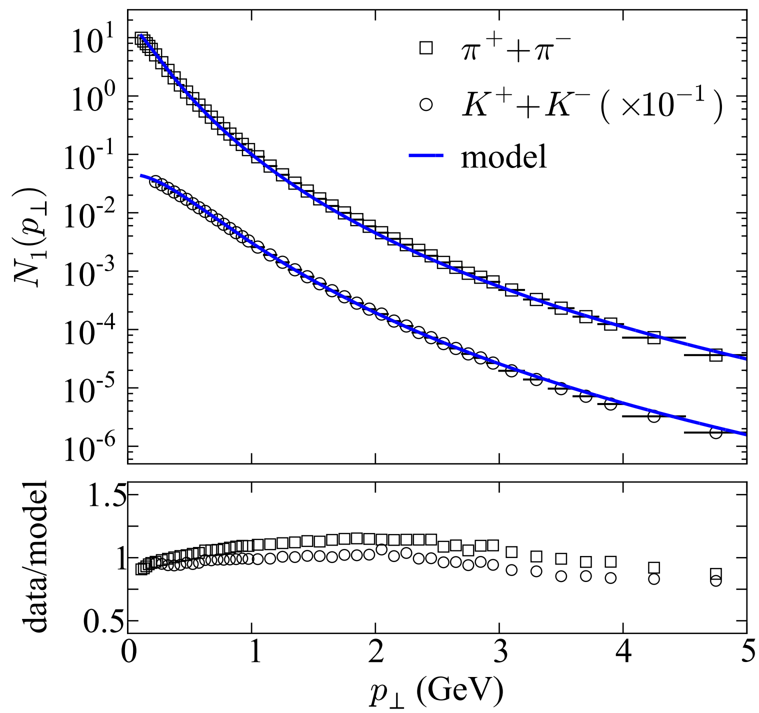

Figure 1: The single particle distributions for pions and kaons as measured by ALICE in p+p collisions at $\sqrt{s} = 2.76$ TeV, compared to the statistical cluster model with $T = 140$ MeV and $\kappa = 5$. The ratios data/model are shown at the bottom.

and

$$dN_2^{(2c)} = \int du_1 \int du_2 W(u_1, u_2) dN_1(p_1; u_1) dN_1(p_2; u_2), \tag{4}$$

where $W(u)$ is the distribution of (four)velocity of a cluster and $W(u_1, u_2)$ is the corresponding distribution of two clusters[3]. For the two particle correlation function

$$C(p_1, p_2) d^2 p_{1\perp} dy_1 d^2 p_{2\perp} dy_2 \equiv dN_2(p_1, p_2) - dN_1(p_1) dN_1(p_2), \tag{5}$$

[3] $\int W(u)du = \langle N \rangle$; $\int du_1 du_2 W(u_1 u_2) = \langle N(N-1) \rangle$ where N denotes number of clusters.

with $dN_1(p) = \int du W(u) dN_1(p,u)$, we thus have

$$C(p_1,p_2) d^2p_{1\perp} dy_1 d^2p_{2\perp} dy_2 = \\ = dN_2^{(1c)}(p_1,p_2) + \int du_1 \int du_2 C_u(u_1,u_2) dN_1(p_1;u_1) dN_1(p_2;u_2). \qquad (6)$$

where

$$C_u(u_1,u_2) = W(u_1,u_2) - W(u_1)W(u_2) \qquad (7)$$

is the two-cluster correlation function.

If clusters are independent, i.e., $C_u(u_1,u_2) = 0$, we have

$$C(p_1,p_2) d^2p_{1\perp} dy_1 d^2p_{2\perp} dy_2 = dN_2^{(1c)}(p_1,p_2). \qquad (8)$$

Consider a cluster[4] at rapidity Y moving in the transverse direction with the velocity $v_\perp$. We have

$$u_0 = \gamma_\perp \cosh Y; \quad u_z = \gamma_\perp \sinh Y; \quad u_\perp = \gamma v_\perp; \quad v_z = \tanh Y. \qquad (9)$$

and the formula (1) becomes

$$dN_1(p,u) = e^{-\beta\gamma_\perp m_\perp \cosh(y-Y) + \beta p_\perp u_\perp \cos(\phi_u - \phi)}, \qquad (10)$$

where ϕ_u and ϕ are the azimuthal angles of the cluster and of the produced particle, respectively.

Following [7] we take

$$W(u)du \sim \gamma_\perp^{-\kappa} d\gamma_\perp d\phi_u G(Y) dY, \qquad (11)$$

where $G(Y)$ is the distribution of clusters in rapidity.[5]

2.1 Correlation in rapidity and azimuthal angle

We start with the correlations in rapidity and azimuthal angle. Using the formulae of the previous section we have

$$dN_2^{(1c)} = dy_1 d^2p_{1\perp} dy_2 d^2p_{2\perp} \int \gamma_\perp^{-\kappa} d\gamma_\perp \int d\phi_u dY G(Y) \\ e^{-\beta\gamma_\perp[m_{1\perp}\cosh(y_1-Y) + m_{2\perp}\cosh(y_2-Y)]} e^{\beta u_\perp [p_{1\perp}\cos(\phi_u-\phi_1) + p_{2\perp}\cos(\phi_u-\phi_2)]}. \qquad (12)$$

---

[4]Henceforth we assume that clusters are uncorrelated.

[5]We note that our results do not depend on the specific shape of $G(Y)$.

To obtain the distributions of $y_1 - y_2 \equiv \Delta y$ and $\phi_1 - \phi_2 \equiv \Delta\phi$ at fixed $p_{1\perp}$ and $p_{2\perp}$ we integrate over ϕ_u, $y_+ = y_1 + y_2$, $\phi_+ = \phi_1 + \phi_2$ and Y. The result is

$$C(\Delta y, \Delta\phi) \sim \int \frac{d\gamma_\perp}{\gamma_\perp^\kappa} K_0\left[\beta\gamma_\perp D_m(\Delta y)\right] I_0[\beta u_\perp D_p(\Delta\phi)], \tag{13}$$

where I_0 and K_0 are the modified Bessel functions of the first and second kind, respectively, and

$$\begin{aligned} D_m(\Delta y) &= \sqrt{m_{1\perp}^2 + m_{2\perp}^2 + 2m_{1\perp}m_{2\perp}\cosh(\Delta y)}, \\ D_p(\Delta\phi) &= \sqrt{p_{1\perp}^2 + p_{2\perp}^2 + 2p_{1\perp}p_{2\perp}\cos(\Delta\phi)}. \end{aligned} \tag{14}$$

Correlations in $\Delta\phi$ at fixed $p_{1\perp}$ and $p_{2\perp}$ can be obtained by integrating independently y_1 and y_2 with the result[6]

$$C(\Delta\phi) \sim \int \frac{d\gamma_\perp}{\gamma_\perp^\kappa} K_0[\beta\gamma_\perp m_{1\perp}] K_0[\beta\gamma_\perp m_{2\perp}] I_0\left[\beta u_\perp D_p(\Delta\phi)\right]. \tag{15}$$

For fixed Δy we have, similarly,

$$C(\Delta y) \sim \int \frac{d\gamma_\perp}{\gamma_\perp^\kappa} K_0\left[\beta\gamma_\perp D_m(\Delta y)\right] I_0[\beta u_\perp p_{1\perp}] I_0[\beta u_\perp p_{2\perp}]. \tag{16}$$

2.2 Correlation in transverse momentum

It is also interesting to consider the distribution of moduli of transverse momenta $[p_{1\perp}, p_{2\perp}]$. Integrating $C(p_1, p_2)$ over $y_1, y_2, \phi_1, \phi_2, Y, \phi_u$, one obtains

$$C(p_{1\perp}, p_{2\perp}) \sim \int \frac{d\gamma_\perp}{\gamma_\perp^\kappa} K_0[\beta\gamma_\perp m_{1\perp}] K_0[\beta\gamma_\perp m_{2\perp}] I_0[\beta u_\perp p_{1\perp}] I_0[\beta u_\perp p_{2\perp}]. \tag{17}$$

3 Results

As explained in the introduction, the two parameters of the model: the temperature in the cluster decay $T = 140$ MeV and the power $\kappa = 5$ were determined from the fit to the pion and kaon single-particle transverse momentum distributions measured by ALICE [17].

[6] We skip the factors $[2\pi - \Delta\phi]$ (for $\Delta\phi < \pi$) and $[\Delta\phi]$ (for $\Delta\phi > \pi$) since they are canceled when $C(\Delta\phi)$ is divided by the distribution of mixed events.

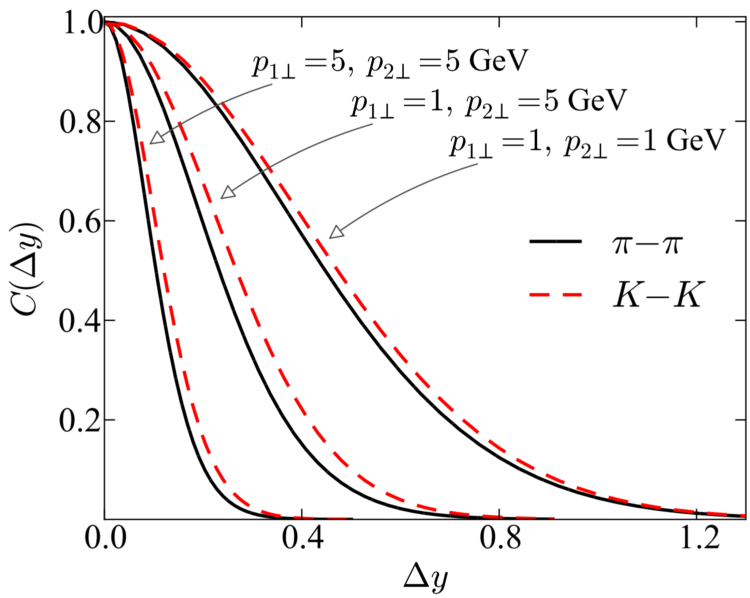

Figure 2: The two-particle correlation function (16) from a statistical cluster for pairs of pions and kaons with various values of the transverse momenta, plotted vs. $\Delta y = y_1 - y_2$, the rapidity separation between the two particles. $T = 140$ MeV and $\kappa = 5$. C is scaled to 1 at $\Delta y = 0$.

The correlation functions in azimuthal angle and rapidity, given by (15) and (16), are shown in Figs. 2, 3. In Fig. 2 the correlation function $C(\Delta y; p_{1\perp}, p_{2\perp})$ (normalised to 1 at $\Delta y = 0$) is plotted vs $\Delta y = |y_1 - y_2|$, for pairs of pions and kaons, at various values of the transverse momenta. One sees that C gets narrower with increasing $p_{1\perp}$ and $p_{2\perp}$ and there is also some mass dependence. The correlation function $C(\Delta\phi; p_{1\perp}, p_{2\perp})$ is plotted in Fig. 3. Similar features are also seen, except that the dependence on particle mass is more pronounced.

Numerical calculations show that for sufficiently high transverse momenta (above ~ 2 GeV) and vanishing particle masses, the two-particle correlation functions in rapidity and in azimuthal angle, can be approximated by Gaussians with the width squared proportional to T^2 and inversely proportional to the product $p_{1\perp}p_{2\perp}$. The proportionality factor is close to 2κ.

Recently the CMS Collaboration published [19] extensive studies of the two-particle azimuthal correlation functions in p+Pb collisions at $\sqrt{s} = 5.02$ TeV. In Fig. 4 they are compared with the predictions of the statistical

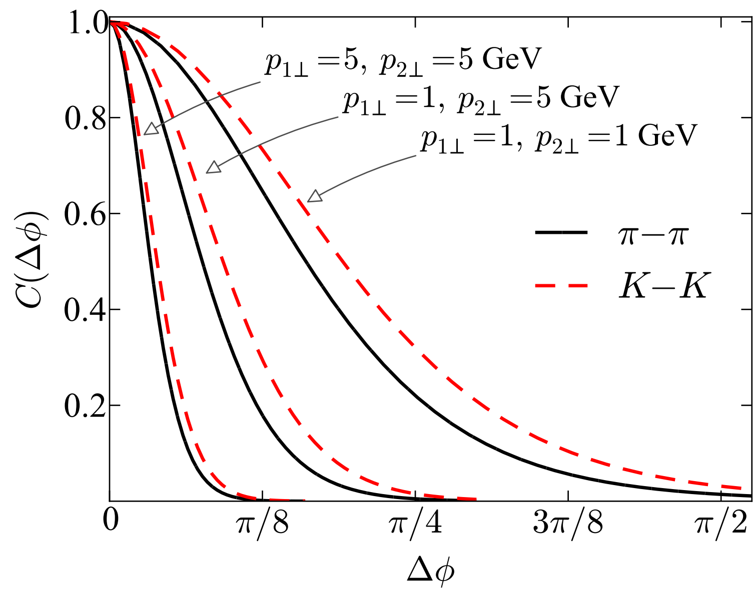

Figure 3: The two-particle correlation function (15) from a statistical cluster for pairs of pions and kaons and with various values of the transverse momenta, plotted vs. $\Delta\phi = \phi_1 - \phi_2$, the relative azimuthal angle between the two particles. In this calculation $T = 140$ MeV and $\kappa = 5$. C is scaled to 1 at $\Delta\phi = 0$.

cluster model. One sees that the data are reasonably close to the model predictions at transverse momenta in the region of $1 - 2$ GeV. At higher transverse momenta the model gives correlation functions which seem somewhat too narrow.[7]

This result agrees with the idea that the model is applicable only at relatively small transverse momenta. On the other hand it may perhaps also indicate the presence of cluster-cluster correlations (described by the second term in the R.H.S. of (6)) at transverse momenta above 2 GeV. Possible resolution of this dilemma would require more sophisticated studies and goes beyond the scope of this paper.

It would be also interesting to study balance functions [20–22]. This is not possible at the present stage of the model since it requires additional

[7] The published CMS data [19] are modified by other physical effects, e.g., flow in p+Pb, the back-to-back peak in $\Delta\phi$ (which is not considered in the present paper), and by the procedure of the background removal.

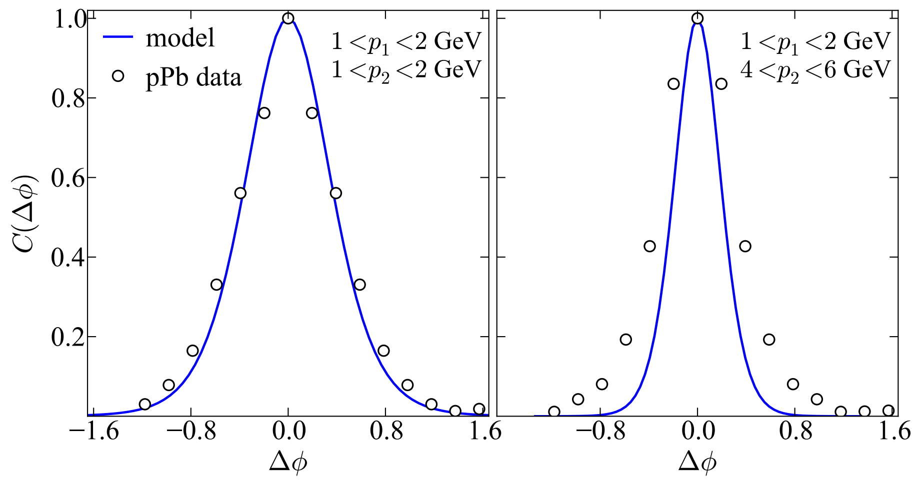

Figure 4: The two-particle correlation function from a statistical cluster for low (left) and high (right) transverse momenta compared with the CMS data [19] on the short-range azimuthal correlation function measured in p+Pb collisions at $\sqrt{s} = 5.02$ TeV. $T = 140$ MeV, $\kappa = 5$. All functions are scaled to 1 at $\Delta\phi = 0$.

information on the distribution of the cluster charges.

The transverse momentum correlation (17), divided by the product of the two single-particle distributions[8],

$$c(p_{1\perp}, p_{2\perp}) = \frac{C(p_{1\perp}, p_{2\perp})}{N_1(p_{1\perp})N_1(p_{2\perp})} \tag{18}$$

is shown in Figs. 5 and 6. $c(p_{1\perp}, p_{2\perp})$, normalized to 1 at $p_{1\perp} = p_{2\perp}$, is plotted in Fig. 5 vs. the ratio $|p_{1\perp} - p_{2\perp}|/p_{1\perp}$ (with $p_{2\perp} \leq p_{1\perp}$) for various values of $p_{1\perp} = 1, 1.5, 5$ GeV.[9] In Fig. 6 the value of c at $p_\perp = p_{1\perp} = p_{2\perp}$ is plotted vs. $p_\perp$. One sees a rather fast increase of c with increasing $p_\perp$.

4 Summary and comments

In summary, we have constructed the two-particle correlation functions induced by the decay of a statistical cluster. The explicit formulae were given

[8]It is convenient to use this definition of $c(p_{1\perp}, p_{2\perp})$ since it is proportional to the number of pairs divided by the number of pairs in mixed events.

[9]We checked that above $p_{1\perp} = 5$ GeV the curves practically do not change any more.

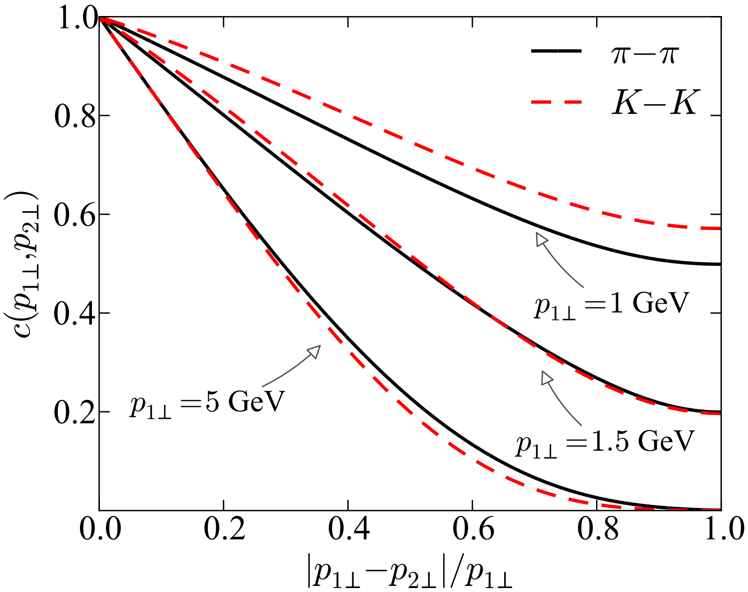

Figure 5: The transverse momentum correlation function (18) from a statistical cluster for pairs of pions and kaons vs. $(p_{1\perp} - p_{2\perp})/p_{1\perp}$, for various values of $p_{1\perp}$. Here $T = 140$ MeV and $\kappa = 5$. c is scaled to 1 at $p_{1\perp} - p_{2\perp} = 0$.

for correlations in rapidity, azimuthal angle and in transverse momentum. Using the parameters of the model determined from the fit to the single-particle distributions ($\kappa = 5,\ T = 140$ MeV) the correlation functions were evaluated. Qualitative comparison with the CMS data on azimuthal correlations in p+Pb collisions at $\sqrt{s} = 5.02$ TeV shows that the model works well at transverse momenta around $1 - 2$ GeV. For larger transverse momenta the evaluated correlation function looks somewhat too narrow, possibly indicating presence of additional inter-cluster correlations.

Several comments are in order.

(i) It should be emphasised that the present calculation follows directly from the statistical cluster model and thus contains no free parameters: the value of the freeze-out temperature ($T \sim 140$ MeV) and the parameter $\kappa = 5$ were determined from the single-particle transverse momentum distribution of pions and kaons. It seems remarkable that spectra of both pions and kaons can be described simultaneously with exactly the same cluster distribution.

(ii) As already explained in the Introduction, we hypothesize that at the final stage of the production process, the statistical clusters are formed and

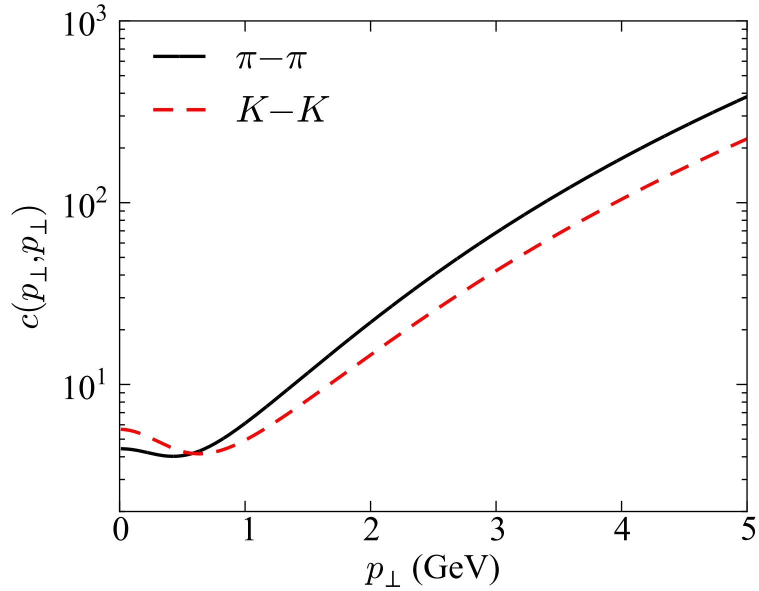

Figure 6: The transverse momentum correlation function (18) from a statistical cluster for pairs of pions and kaons vs. $p_\perp = p_{1\perp} = p_{2\perp}$. $T = 140$ MeV and $\kappa = 5$.

decay into observed particles. At high transverse momenta, jet physics is expected to induce correlations between clusters and thus additional correlations between produced particles. Detailed experimental investigation of this region could therefore verify universality of the cluster hypothesis and may also give useful information on the structure of jets.

(iii) It would be also most interesting to measure and compare the short-range correlation functions in p+p and e^+e^- collisions in order to test universality of the statistical cluster picture of particle production. Also measurement of correlations for various pairs of particles can be very useful in this respect.

(iv) In our calculations we have ignored the correlations which may appear in the cluster decay. The statistical clusters are rather special objects and their physical interpretation, and consequently the nature of their internal correlations, is (in our opinion) not clear. For example, it is not obvious if clusters, representing multi-particles states, have a well-determined mass. The detailed comparison of the model with data in p+p or e^+e^- collisions should shed more light on these questions.

(v) In the present paper we have not discussed the baryon production, as it is not clear if at LHC energies the statistical model can describe correctly the baryon multiplicities. Within the statistical cluster model one may overcome this difficulty, e.g., by postulating that the clusters emitting baryons are of different nature than those producing mesons only. This problem is under investigation.

Acknowledgements

We thank Kacper Zalewski for very useful discussions. This investigation was supported by the Ministry of Science and Higher Education (MNiSW), by founding from the Foundation for Polish Science, and by the National Science Centre (Narodowe Centrum Nauki), Grant Nos. DEC-2013/09/B/ST2/00497 and DEC-2014/15/B/ST2/00175.

References

[1] There exists enormous literature on this subject. For a review, see e.g. W. Kittel and E. A. De Wolf, Soft multihadron dynamics (World Scientific, 2005)

[2] A selection of some relevant works includes: A. Bialas, K. Fialkowski and K. Zalewski, Phys. Lett. B39 (1972) 211; F. Hayot and A. Morel, Nucl. Phys. B68 (1974) 323; Acta Phys. Pol. B4 (1973) 941; A. Morel and G. Plaut, Nucl. Phys. B78 (1974) 541; F. Hayot, F.S. Henyey and M.Le Bellac, Nucl. Phys. B80 (1974) 77.

[3] F. Becattini, arXiv:0901.3643 [hep-ph].

[4] F. Becattini, hep-ph 9701275; published in Proc. of XXXIII Eloisatron Workshop, Erice 1996, p.74; F. Becattini and U. Heinz, Z. Phys. C76 (1997) 269.

[5] A. Andronic, P. Braun-Munzinger and J. Stachel, Nucl. Phys. A772 (2006) 167; J. Stachel, A. Andronic, P. Braun-Munzinger and J. Redlich, J. Phys. Conf. Ser. 509 (2014) 012019; J. Letessier, J. Rafelski, A. Tounsi, Acta Phys. Pol. B27 (1996) 1037; J. Letessier and J. Rafelski, Phys. Rev. C59 (1999) 947.

[6] F. Becattini, P. Castorina, A. Milov, H. Satz, Eur. Phys. J. C66 (2010) 377; L. Ferroni and F. Becattini, Acta Phys. Polon B43 (2012) 571; Eur. Phys. J. C71 (2011) 1824; F. Becattini, J. Manninen and M. Gazdzicki, Phys. Rev. C73 (2006) 044905.

[7] A. Bialas, Phys. Lett. B747 (2015) 190.

[8] C. Tsallis, J. Stat. Phys. 52 (1988) 479.

[9] C.-Y. Wong and G. Wilk, Acta Phys. Pol. B43 (2012) 2047; Phys. Rev. D87 (2013) 114007.

[10] M.D. Azmi and J. Cleymans, J. Phys. G41 (2014) 065001; Eur. Phys. J. C75 (2015) 430.

[11] C. Y. Wong, G. Wilk, L. J. L. Cirto and C. Tsallis, Phys. Rev. D **91**, no. 11, 114027 (2015).

[12] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. D83 (2011) 052004; Phys. Rev. C83 (2011) 064903.

[13] K. Aamodt *et al.* [ALICE Collaboration], Phys. Lett. B693 (2010) 53; Eur. Phys. J. C73 (2013) 2662.

[14] G. Aad *et al.* [ATLAS Collaboration], New J. Phys. 13 (2011) 053033.

[15] V. Khachatryan *et al.* [CMS Collaboration] JHEP 1002 (2010) 041; Phys. Rev. Lett. 105 (2010) 022002.

[16] G. Wilk and Z. Wlodarczyk, Phys. Lett. A379 (2015) 2941.

[17] B. B. Abelev *et al.* [ALICE Collaboration], Phys. Lett. B **736**, 196 (2014).

[18] S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1108**, 086 (2011).

[19] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **724**, 213 (2013).

[20] D. Drijard *et al.* [ACCDHW Collaboration], Nucl. Phys. B155, 269 (1979); B166, 233 (1980). C. Alt *et al.* [NA49 Collaboration], Phys. Rev. C71, 034903 (2005); C76, 024914 (2007).

[21] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. 90, 172301 (2003); B.I. Abelev et al. [STAR Collaboration], Phys. Lett. B690, 239 (2010); M.M. Aggarwal *et al.* [STAR Collaboration], Phys. Rev. C82, 024905 (2010); B. Abelev *et al.* [ALICE Collaboration], Phys. Lett. B723, 267 (2013); J. Adam *et al.* [ALICE Collaboration], Eur. Phys. J. C76 (2016), 86.

[22] S.A. Bass, P. Danielewicz and S. Pratt, Phys. Rev. Lett. 85, 2689 (2000); A. Bialas, Phys. Lett. B579, 31 (2004).